Final Evaluation Report

Mathematical Reasoning with Connections

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Prepared for Riverside County Office of Education

By Impact Research Consulting

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Dissemination of this impact report will be conducted via RCOE's website (full report). The report will also be submitted to ERIC database.

The findings and conclusions in this report do not necessarily represent the official positions or policies of the funders.

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Executive Summary

The MRWC course is a 4th year advanced mathematics course developed by a consortium of mathematics professors and math educators from CSU, UC, and CCC higher education systems, together with mathematics specialists from County Offices of Education and local school districts. It has been specifically designed to address the need for stronger mathematics preparation for transitioning from high school to college and career pathways and reduce the need for students to enroll in remedial mathematics courses upon entering college. Pilot study of the MRWC course that was conducted in 2017-18, while the curriculum was being developed compared the participating students' performance on the Grade 11 Mathematics Early Assessment Program assessment items to the performance of students in traditional advanced mathematics courses (precalculus, statistics and probability, integrated mathematics 4, etc.), where students were not exposed to MRWC. The results of the pilot study suggested that the MRWC curriculum can potentially improve student mathematics achievement and help reduce the need for remediation at the college level. With this understanding, RCOE applied for the i3 development grant to support the development, implementation, and evaluation of the MRWC course curriculum, strategies, and professional learning model. The Investing in Innovation (i3) development grant was awarded to RCOE in 2016 (Dev113) to support providing extensive, in-depth professional learning opportunities for collaborative teams of secondary educators designed to prepare students for placement into college-level courses in mathematics, without the need for remediation. Illuminate Education and Impact Research Consulting were contracted to conduct a rigorous independent evaluation of the impact of the MRWC on student achievement and to assess the fidelity of implementation of the MRWC across Inland Empire.

The MRWC Program consists of the three main components: advanced mathematics curriculum and strategies, MRWC student and teacher materials and professional learning model. These components were combined into a comprehensive program intended to lead to instruction, classroom practices and teacher collaboration that equip students with strategic and flexible mathematical thinking as well as to enable them to become self-reflective learners. It focuses on commonalities between and among all high school mathematics topics, so that students come to understand the entire breadth of high school mathematics curriculum, discuss, and analyze alternative solutions for problems to enhance flexibility with the applications of procedures and engage in classroom activities designed to model and foster real life applications of the advanced mathematical concepts. The MRWC professional learning component included providing 20 days (14 pre-implementation and 6 throughout the implementation year) of professional development and training and support to intervention (i3) teachers in planning and implementing powerful mathematics instruction across the curriculum.

The executive summary presents the findings of the impact evaluation analysis, assessment of fidelity implementation and concludes with the short overview of the discussion and implications. The outcome measures used in the evaluation of the impact of the MRWC Program was student achievement in advanced mathematics measured by Precalculus Concept Assessment (PCA). The confirmatory research question used in the impact evaluation was as follows:

1. What is the impact, on high students' mathematics achievement as measured by the PCA, of one year of exposure to MRWC (confirmatory)?

The main research questions related to fidelity of implementation were:

1. What proportion of study teachers are implementing the MRWC course with adequate fidelity as measured by MRWC teacher implementation logs and reflections and coaching observation forms?
2. What proportion of MRWC teachers received coaching observations at least three times during the implementation year (once per theme)?
3. What proportion of MRWC teachers attended at least 13 out of the 15 required pre-implementation professional development days?
4. What proportion of MRWC teachers attended at least four out of five days of professional learning and collaboration throughout the implementation year (5 days)?
5. What proportion of study teachers received all of the required curriculum materials (teacher and student) prior to teaching each respective module?

To estimate the impact of MRWC on student mathematics achievement, a quasi-experimental design was selected as the most rigorous design possible given the constraints of equal access and district/site voluntary participation. Forty-five intervention schools with close to 5,500 students met criteria to be included in the final impact analysis study. Nineteen high schools with close to 1,500 students served as our comparison schools (not exposed to the project during their comparison participation year). Students at the intervention schools were matched with similar students at comparison schools, who took other advanced mathematics courses on several demographic variables including ethnicity, gender, and Grade 11 mathematics achievement. One-to-many matching with replacement was done for treatment (i3) and control students, so multiple treatment student could be matched to a given comparison student. The final analytic sample consisted of 5,415 MRWC students and 5,393 comparison students. After the matching was conducted, all matched students were included in a regression model that included as covariates the same variables that were used in the matching process.

Compared to the non-MRWC students, students in MRWC schools scored higher on PCA, and that was significant at the 1% level. MRWC was found to have a positive impact on students' mathematics achievement, and that this impact was unlikely to have happened by

chance. The estimated standardized effect size was also calculated for the impact estimate. The effect size of MRWC was 0.31 standard deviations, which can be considered a small but significant positive effect, especially in the context of a broad, curriculum-based intervention implemented in the educational settings.

The results of the fidelity of implementation analysis indicated that all five of the intervention components were implemented with fidelity in all three of the implementation study years (2018-19, 2020-21 and 2021-22). Those intervention components were curriculum implementation, participation in the professional learning pre- and during implementation, coaching observations, collaborations and MRWC material delivery. Teacher participation in professional learning was implemented with fidelity by at least 85% of the teachers in all three years. Coaching was received by 100% of the teachers in all three years. Materials were delivered on time, prior to each of the theme implementation to 100% of the teachers in all three years. Finally, at least 80% of the participating teachers delivered at least 85% of the curriculum they taught with high fidelity in each of the implementation study years.

Overall, all of the implementation study components were found to be completed with high fidelity in all three of the implementation years. The impact evaluation analysis found that MRWC produced a small but significant positive impact on student mathematics achievement, with the small effect size of 0.31 (31% of standard deviation). It should be noted that considering that MRWC is a broad, and in many cases site-wide curriculum intervention, this effect size was comparable, if not slightly higher than the average effect sizes produced by the studies of district-wide curriculum interventions reviewed by Lipsey et al. (2012). Additionally, the participating Inland Empire teachers, administrators and counselors consistently shared with the project leadership that both teachers and students involved definitely enjoyed many benefits of the program and that this intervention still has a potential to improve students' mathematics achievement, especially if students are exposed to these strategies as early as possible in their high school mathematics experience. Therefore, future evaluations could assess whether impacts of this pedagogy and professional learning model would be even greater, if teachers were able to implement it throughout all of the high school mathematics courses they teach.

Chapter 1. Introduction

The MRWC Pilot Studies (2016-2018)

The Mathematical Reasoning with Connections (MRWC) project is a collaborative project of multiple partner agencies that include post-secondary institutions, county offices of education and school districts within Riverside and San Bernardino Counties. The primary goal of the project was two-fold: 1) create a new conceptually based senior year mathematics course, and 2) provide extensive, in-depth professional learning opportunities for collaborative teams of secondary educators and their school-site administrators to support the implementation and evaluation of grade 12 experiences that are designed to prepare students for placement into college-level courses in mathematics.

To achieve this goal, the project provided intensive Professional Learning (PL) to high school mathematics teams (teachers, instructional coaches, academic counselors, and administrators) to support successful adoption of the new pedagogy and curriculum content that focused on enhancing key math content, understanding of standards for mathematical practices and CA Common Core State Standards, knowledge and beliefs for teaching math, self-efficacy for teaching math, and pedagogical fluency. Professional Learning was provided over 20 days per year during multi-day institutes, academic year workshops, and local professional learning community (PLC) meetings. Before the project started, the partnership conducted an in-depth needs assessment and analysis of student data and developed an innovative concept that addressed the need to align the high school math curriculum and instruction with college and career expectations. The new senior level math course was conceptualized by the leadership and the curriculum writing team as a potential long-term strategy for significantly reducing the need for investment in developmental coursework at the postsecondary level.

Two pilot implementation studies of the MRWC curriculum and professional learning model were conducted that were concurrent with the curriculum development process. The first pilot evaluated the implementation of Theme 1 (out of a total of three themes) in the Fall of 2016. A total of nineteen teachers participated in a Theme 1 pilot (alpha cohort). The nineteen alpha teachers received 10 days of professional learning in the summer of 2016 and started implementing Theme 1 in their advanced mathematics classes in Fall 2016. No formal impact evaluations have been conducted on the pilot implementation of Theme 1, as it did not represent the entire MRWC intervention. Informal evaluation of Theme 1 implementation by alpha cohort was conducted to provide implementation and student engagement data that later informed the updates to Theme 1, as well as the development of Themes 2-3. Upon implementing Theme 1, alpha teachers completed the implementation evaluation surveys and students were given an assessment based on the topics covered in Theme 1. The surveys asked teachers to reflect on the following:

- course content
- module style and organization
- target audience
- timing
- MRWC comparison with currently used textbooks/materials
- student engagement and motivation throughout the course
- quality of and satisfaction with professional development provided

Out of 19 teachers, who responded to the Theme 1 implementation survey, 16 (84%) thought that the Module they piloted worked well as the first module in MRWC series. Five out of 19, were concerned that the time proposed for implementation of the module (4 weeks) might be insufficient and should be extended. In response to this concern the new allotted time for Theme 1 has been set to 6 weeks. Eighteen teachers (95%) felt that, overall, the content taught in Theme 1 was appropriate for the target audience (high-school seniors, with at least Level 2 SBAC 11 achievement level and with a C or better in Algebra II, Integrated Math III or a similar course), but eight of these teachers also mentioned that some components of the theme might be somewhat challenging for the target audience and required some differentiation and additional time for some students to grasp and master. Only one of the teachers felt that the theme content would be more suitable for an Honors Pre-Calculus or a similar advanced course. The majority of the teachers (17 out of 19) felt that the MRWC materials were at least equally or even more rigorous than their current textbooks/curriculum materials, especially in the level of inquiry and reasoning required from the students. Three of the teachers specifically felt that MRWC curriculum was more aligned to Common Core standards than was their current curriculum. When asked if they think MRWC curriculum can help engage and motivate students, 11 teachers (58%) stated that the amount of discourse-based activities and games embedded in the curriculum would definitely be a motivating factor for the students. However, seven teachers (37%) felt that while some activities might be engaging, others would be difficult for some students, which might lead to struggle and potential disengagement. The majority of the teachers (17 out of 19) felt that the PD provided was very helpful, especially when they could work through the activities in groups. With regards to the time allocated to the PD, most teachers (15 out of 19) felt that it was enough to cover just Theme 1 but that more PD will be needed for the entire course. The information collected from this evaluation was incorporated into the development of Themes 2-3.

In 2016 MRWC course development and implementation research was funded through the California Mathematics Readiness Challenge Initiative (CMRCI) grant. The CMRCI grant was awarded to the California State University, San Bernardino (CSUSB) in partnership with Riverside County Office of Education (RCOE) and participating school districts with an overarching goal of finishing the development of a new conceptually based senior year math course and providing extensive, in-depth professional learning opportunities for collaborative

teams of secondary educators designed to prepare students for placement into college-level courses in mathematics. The second pilot (beta cohort) that included the implementation of the first 2 completed themes of MRWC course took place during 2017-18 school year. Full MRWC professional learning model was implemented for the first time during this pilot. Participating beta cohort teachers and coaches received 5 days of professional development in the Spring of 2017, 10 days of professional development directly preceding the implementation in the summer of 2017 (summer institute) and 5 additional days throughout the implementation year (November – December 2017). Beta teachers implemented Themes 1 and 2, while Theme 3 was still being developed. All of the teachers and students participating in the beta cohort participated in data collection and the following data pieces were collected:

- Senior year mathematics assessment administered to students enrolled in MRWC course and to the comparison students in traditional advanced mathematics courses.
    - In 2017-18 school year, we used the released EAP items from the CA state assessment.
- MRWC student advanced mathematics perception surveys (online, once a year)
- MRWC teacher survey (online, once a year)
- MRWC teacher implementation survey/logs (completed by teachers online, as they are teaching each of the course Themes)
- MRWC coaching logs (completed by coaches online, as they observe MRWC teachers at least once per Theme).
- MRWC principal and counselor surveys (completed by principals and academic counselors online, once a year)

The second pilot study used a comparison design, with multiple measures and multiple sources of data (i.e., student and teacher surveys, teacher attendance of professional development and PD reflections, and student assessment). Attendance at professional development meetings and reflections were also collected for the 2017-18 beta cohort. MRWC and comparison group performance, as well as student and teacher survey results, were collected and analyzed for both summative and formative purposes.

During 2017-18 school year, 40 MRWC beta cohort teachers completed end-of-year teacher surveys. Approximately 1,436 MRWC students and 338 Comparison students completed end-of-year measures (i.e., student EAP assessments and surveys). Comparisons were made between student taught by teachers who participated in the MRWC Professional Development and taught the course, and students taking senior year math courses taught by teachers without any additional support or materials (i.e., “business as usual”). Individual changes on the outcome variables were calculated and group comparisons were made on outcome change. When feasible, independent samples t-test and the Welch approximation t-test (accounts for

unequal variances and unequal sample sizes of the samples) were calculated to determine statistical significance at the .05 probability level.

EAP assessments were administered to the MRWC students after the completion of the course in 2017-18 school year and to the control students after the completion of their respective fourth year math courses. Independent samples t-tests were conducted to compare the MRWC and control student performance on the grant assessments. The assessments were completed by 1,436 MRWC and 338 control students. Average MRWC student performance was about 35.5% correct, while the average control student performance was about 30.5% percent correct. The mean difference was about 5 percentage points. This was a statistically significant difference $t(1,772) = 3.644$, $p = 0.0003$. To account for unequal sample sizes in the MRWC and control samples, an additional t-test (Welch's t-test) was conducted that is robust to unequal sample sizes and unequal variances to determine if the significant finding would hold under the assumption of unequal variances. The Welch's t-test had also suggested a significant difference between the MRWC and control student performance. Overall, it was determined that average MRWC student performance (percent correct on the assessment) was significantly higher for the pilot cohort of students, who were taught by the beta cohort of MRWC teachers and received Theme 1 and 2 of the MRWC curriculum, compared to the matched control students' performance that participated in other advanced mathematics courses (pre-calculus, statistics and probability, integrated math IV and others).

Student Mathematics Perception surveys were conducted with MRWC and control students at the completion of 2017-18 MRWC course. Control student responses included students from Integrated Math 4 (IM4), Statistics and Probability and Pre-Calculus level courses to allow for more appropriate matching of students' senior level math experiences. Responses from students in AP Statistics and AP Calculus courses were not included in the analysis. The survey results indicated that MRWC students, compared to students enrolled in Pre-Calculus, Statistics and Probability and IM4 classes, reported higher levels of enjoying finding multiple solutions for math problems (38.2% MRWC vs 33.5% Control), enjoyed having discussions with classmates about the best ways to solve math problems (67.8% MRWC vs 56.4% Control) and felt that they had a deeper understanding of math vocabulary (54.4% MRWC vs 49.7% Control) after completing the course. A higher percentage of MRWC students felt that they were prepared to enroll in college-level mathematics courses (55% MRWC vs 50.2% Control).

The pilot teacher survey results were also promising. Specifically, PD satisfaction surveys collected at the end of each PD showed that all 40 of the 2017-18 MRWC beta cohort teachers were satisfied with the PD and spoke of its value and usefulness. The teachers specified that they were getting sufficient support as they were teaching the MRWC course and that all of the sessions and planning time were extremely helpful and productive.

Additionally, about 95% of teachers stated that the training provided helped them understand the goals of the project and gave them valuable information to become a more effective math teacher. About 90% felt that they could fully implement the MRWC curriculum strategies in their classroom. Teacher implementation surveys conducted with 2017-18 MRWC pilot teachers in November 2017 (pre) and in May 2018 (post). In November 2017, 79% of teachers believed that MRWC course could help their students be better prepared for and successful in college-level mathematics courses. By May 2018, this rate was up to 94% of teachers. About 82% of teachers at both pre and post survey believed that MRWC was making a substantial difference in their students' mathematics abilities. About 84% of teachers at pre-survey and 86% of teachers at post survey believed that all of their students could benefit from the MRWC course. At pre-survey, about 82% of teachers believed that the MRWC workshops and professional development have given them valuable information on how to become a more effective teacher. At post-survey, over 93% of teachers believed so. The teacher survey results indicated that the majority of the MRWC teachers were integrating various MRWC strategies and techniques into other classes they teach. At post-survey, over 97% of teachers stated that MRWC curriculum and training impacted their teaching in other classes. Specifically, multiple teachers reported that MRWC made them focus more on the connections between different concepts, while teaching Integrated Math series, pre-calculus, and calculus courses. Other teachers mentioned that MRWC taught them the importance of using correct terminology and notations in all of their courses. Teachers also reported that their entire style of teaching changed (regardless of the class they teach), as they now allowed their students more opportunities to be self-sufficient, independent, and reflective in the learning process and learn through group work, explorations, collaborations and reasoning. Several teachers said that MRWC training showed them how to change their teaching for high school level mathematics by consistent incorporation of mathematical discourse and made them more confident in both content and teaching strategies. The PD reflections supplemented these findings, with multiple teachers reporting that they now focus on proper function notations in their Integrated Math 1 (IM 1) courses to build background knowledge, increase discussion time and group work in other classes, incorporate sign charts for the polynomials into Integrated Math 3 (IM3) and Pre-Calculus and use Tarsias in other courses.

This pilot evaluation of the beta cohort implementation was included into the first (2018) i3 annual performance report and helped develop and finalize the rest of the MRWC curriculum. The final curriculum contained three themes: 1) Reasoning with Numbers, 2) Reasoning with Functions and 3) Reasoning with Equivalences. The full i3 implementation of MRWC began in fall of 2018 and was originally scheduled to collect impact data in the following three years: 2018-19, 2019-20 and 2020-21. Due to the state-wide school closures and subsequent online learning at all of our participating districts, the project leadership and evaluation team was not able to collect the end-of-year assessment or survey data from the students in 2019-20

school year and include that student and teacher cohort into the impact evaluation. To ensure that three years of impact data are collected and analyzed the project continued its impact data collection in 2021-22 school year after being granted a no-cost extension for an additional year.

The MRWC Investing in Innovation (i3) Development Grant (Dev113)

In December 2016, the Riverside County Office of Education (RCOE), in partnership with the California State University San Bernardino, California State University Long Beach, California State Polytechnic University, Pomona, University of California, Riverside and Riverside Community College received an Investing in Innovation (i3) development grant from the U.S. Department of Education to develop and begin the implementation of the Mathematical Reasoning with Connections (MRWC) course. The MRWC course is a 4th year advanced mathematics course developed by a consortium of mathematics professors and math educators from CSU, UC, and CCC higher education systems, together with mathematics specialists from County Offices of Education and local school districts. It has been specifically designed to address the need for stronger mathematics preparation for transitioning from high school to college and career pathways and reduce the need for students to enroll in remedial mathematics courses upon entering college. MRWC was designed for any student who earns a minimum grade of C in Integrated Math 3 or Algebra 2. This includes SBAC Conditional (Level 3) students who do not necessarily intend to pursue calculus. These students need a 4th year course to fulfill the college readiness requirement and are seeking an option other than statistics. It also includes SBAC Not Ready (Level 2) students who are looking to improve their chances of successfully passing college and university placement exams. MRWC also provides a good option for SBAC Ready (Level 4) students who plan to continue studies in mathematics into calculus but are looking to consolidate and strengthen foundational skills in a 'bridge' course before entering pre-calculus and/or calculus. The MRWC provides a bridge into multiple college and career options, including STEAM, CTE, and non-technical pathways. Students successfully completing MRWC will have acquired content skills and attitudes towards learning that will be expected in entry-level college mathematics. MRWC was created to address the full scope of advanced mathematical topics in a way that is substantively different from the traditional curriculum. Based on the Common Core State Standards viewpoint that mathematics is a cohesive and connected body of work, the MRWC is structured to highlight overarching themes in mathematics that are intrinsic to and underlie many topics in the high school curriculum.

The i3 development grant funded Riverside County Office of Education to implement and evaluate the impact of the Mathematical Reasoning with Connections (MRWC) course for preparing students to enter college-level mathematics courses without remediation. The main question examined in this evaluation study was as follows: What is the effect of MRWC on

the college mathematics readiness skills of grade 12 students as measured by Precalculus Concept Assessment compared to business-as-usual senior year mathematics courses (e.g., Precalculus, Statistics and Probability and Integrated Math 4)? This evaluation examined implementation, effectiveness, periodic progress, and was used to assist in the process of making data-driven decisions for program modification of the i3 supported Mathematical Reasoning with Connections course over the five years of the grant. Once the curriculum was finalized, the first full implementation i3 cohort of teachers joined the program in January 2018, were trained on MRWC curriculum and began implementation in fall 2018. The i3-supported impact and implementation studies were conducted using data collected from the following MRWC cohorts 2018-19, 2020-21, 2021-22.

Chapter 2. The Mathematical Reasoning with Connections Course

This chapter provides a detailed description of the MRWC program as implemented in Riverside and San Bernardino County districts throughout the i3 grant. It includes the discussion of the MRWC theory and curriculum used by the program, and its professional learning model. Table 1 outlines the logic model used in the evaluation. It shows the program inputs, activities and instructional practices as well as expected short-term and long-term outcomes of the program. In addition to outlining the key inputs and outputs as well as describing the intended short- and long-term outcomes, the logic model illuminates the course's underlying rationale – many students enter college without sufficient mathematics skills to be able to succeed in college-level mathematics without remediation. The main goal of the MRWC is to mitigate this issue by equipping high school students with strategic and flexible mathematical thinking, deepen their understanding of and familiarity with mathematical concepts and procedures and help them understand and appreciate the entire breadth of the high school mathematics curriculum to make them better prepared for the college level mathematics courses. Students successfully completing MRWC will have acquired content skills and attitudes towards learning that will be expected in entry-level college mathematics.

The program inputs were professional learning, teacher collaboration support, coaching support as well as MRWC theory, strategies, and curriculum (Themes 1-3) and respective teacher and student materials. These inputs led to activities targeted to influence instructional practices including focusing on commonalities between and among all topics, so that students come to understand the entire breadth of high school mathematics curriculum, discuss and analyze alternative solutions for problems to enhance flexibility with the applications of procedures and engage in classroom activities designed to model and foster real life applications of the advanced mathematical concepts.

These instructional practices were expected to result in several short- and long-term outcomes. Among short-term outcomes were increased level of pedagogical skill, math teaching efficacy and flexibility for teachers and coaches, higher understanding of standards of mathematical practices for administrators and counselors, as well as increased mathematical competency and efficacy and higher learning motivation and improved student attitudes and perception toward advanced math for students. Projected long-term outcomes included teachers utilizing the new pedagogy across all of their courses, students demonstrating college-ready math competency and being successful in their freshman year college-level math courses. The remainder of this chapter further describes the MRWC program inputs.

MRWC Theory

The MRWC's overarching goal is to equip students with strategic and flexible mathematical thinking as well as to enable them to become self-reflective learners. The Mathematical Reasoning with Connections (MRWC) curriculum is intended to help students to deepen their understanding of and familiarity with mathematical concepts and procedures that they have previously encountered. Commonalities and similarities between and among all topics are highlighted so that students come to understand and appreciate the entire breadth of the high school mathematics curriculum as a cohesive body of knowledge. By asking questions that – when answered –generate the need for numbers, operations, properties, concepts, and procedures in high school mathematics, the MRWC course challenges students to approach learning and doing mathematics through a different lens.

Several key principles underlie the Mathematical Reading with Connections course curriculum and its materials:

- Connectedness between overarching themes in mathematics that underlie many topics in high school curriculum.
- Interrelated nature of procedural and conceptual knowledge as well as problem solving and reasoning abilities highlighted throughout the MRWC curriculum.
- Emphasis on discussion and analysis of alternative representations and multiple perspectives for approaching and understanding content to enhance flexibility and fluidity with the applications of procedures.
- Classroom activities designed to model and foster real life applications of the advance mathematical concepts.
- Topics and activities that promote exploratory and collaborative student engagement.
- Modular format of curricular materials that allows for flexible implementation in various settings or replacing existing curriculum as well as responding to the varied needs of students.
- Alignment to the Common Core Standards for Mathematical Practice.

MRWC curriculum is organized in themes, with each theme dedicated to a specific overarching topic, that are designed to enable students to see mathematics as an integrated and cohesive body of conceptual understanding and procedural knowledge:

Theme 1 – Reasoning with Numbers

Reasoning with Integers
Reasoning with Rational Numbers
Reasoning with Irrational Numbers
Reasoning with Real Numbers
Reasoning with Complex Numbers

Theme 2 – Reasoning with Functions

Reasoning with Relationships

Reasoning with Features of Functions

Reasoning with Representations of Functions

Reasoning with Families of Functions

Theme 3 – Reasoning with Equivalences

Exploring Equivalence

Reasoning with Equivalent Expressions

Reasoning about Statements

The MRWC Curriculum and Strategies

The MRWC Curriculum covers all of the optional + (plus) standards for Integrated Mathematics 3 or Algebra 2 and most of the standards included in the curriculum in the CA Mathematics Framework for Precalculus. The MRWC includes three themes: Reasoning with Numbers, Reasoning with Functions and Reasoning with Equivalences. Teachers are expected to teach all three themes throughout the span of the school year. Themes are taught in a sequence, with each theme building upon the knowledge learned in the prior theme. The MRWC curriculum materials include an extensive and detailed teaching manual that provides both mathematical and pedagogical content guidance to strengthen teachers' mathematical understanding and instructional practices. Each theme consists of several sections that are connected by a set of overarching questions that connect the sections. Each section comes with a number of activities. Also, embedded into each section are additional resources and recommendations needed to scaffold less-prepared students' prerequisite knowledge for MRWC, an assessment item bank to promote ongoing evaluation of student progress toward learning objectives, and detailed guides for incorporating technology as an instructional tool. MRWC uses a non-traditional instructional approach emphasizing collaboration and exploration through mathematical activities, problem posing, and the use of technology that will address diverse learning styles. Instruction is designed to challenge students to approach mathematics as sense-making through a focus on questioning and probing deeper. Teacher-led instruction and student explorations focus on discovering the conceptual basis for standard procedures. The MRWC course facilitates the development of students' ability to choose strategically among multiple solutions options, and to articulate the reasons for those decisions. Students use informal and formal justifications to defend their understanding and critique the reasoning of others. Instruction also emphasizes the use of and fluency in the full range of the language of mathematics and notation. Content topics are approached through six instructional modalities i.e., verbal, numeric, symbolic, graphical, geometric, and technological. Different forms of formative and summative assessments are used throughout the course. Students are given multiple opportunities to demonstrate their ongoing conceptual understanding and procedural fluency through mathematical activities, small group discussions and explorations, personal reflections, quick

writes, in addition to worksheets and individual written assessments such as quizzes, tests, final interim and summative exams. Students are also assessed through group projects and oral and written presentations.

The MRWC Materials

The MRWC materials include the following components for each Theme:

- MRWC curriculum binders for teachers and students (1 per theme)
- MRWC student game sets
- MRWC lesson planning templates for each participating district/site team.
- MRWC assessment items

The MRWC Professional Learning Model Components

In addition to the mathematics curriculum and materials, the MRWC program had a strong professional learning model. The primary professional development goal for the program is to support MRWC (i3) teachers in planning and implementing powerful mathematics instruction. To do this, the MRWC management team provides participating teams with a comprehensive staff professional learning program that is paced according to the instructional needs and the learning of accompanying curriculum components necessary for instruction, assessment, and communication of information. To support teachers and coaches in more frequent and effective implementation of the MRWC program, staff development sessions are designed using a combination of a teach/practice model and a coach/mentor model.

During the initial four days of professional learning led by the MRWC curriculum development team, scheduled for spring preceding the first implementation year, teachers and coaches receive an overview of the MRWC course and its key principles and familiarize themselves with Theme 1 subsets. Some of the topics teachers and coaches discuss during these meetings include Mathematical Literacy, Flexibility with Numbers and Operations, Motivating and Locating Rational Numbers, Structure and Fluency in the Rationals, Mathematics from a Different Perspective, Developing Number Fluency and Number Sense, Lesson Planning for Integers and Rationals, Orchestrating Productive Mathematics Discussion and Constructing Products and Quotients of Real Numbers among others. Part of each PD Day is also set aside for site-wide planning using the MRWC planning guides developed by the MRWC curriculum team. Principals and counselors attend Day 1 of the initial MRWC training and participate in discussions around steps needed for MRWC site implementation, getting students enrolled in MRWC courses. A 10-day MRWC summer institute focuses on Themes 2-3 and further team planning and is to be attended by teachers and mathematics coaches in the summer before the first implementation year.

The second component of the professional learning is participation is five days of professional learning and collaboration meetings throughout the first implementation year, which provides

opportunities for teachers and coaches to share their implementation highlights and best practices and get feedback from other teams implementing MRWC on effective ways to handle some of the challenges. During these implementation year collaboration experiences teachers and coaches are encouraged to share effective instructional strategies and student success stories, identify student needs and collaborative work on designing effective strategies for differentiating the MRWC curriculum for students with various needs and identify goals for the upcoming modules.

The third component of professional development is coaching observations and collaborations. District- and site-based coaches or mathematics specialists from the county offices of education work collaboratively with MRWC teachers at their respective sites throughout their first implementation year. Coaches participate and collaborate with teachers on pre-planning, implementation, and post-implementation debriefing. Coaches also observe teachers implementing the curriculum and provide feedback to help improve the instruction, develop lesson plans, discuss strategies to engage students and facilitate classroom discussion. Coaches and teachers participate in at least one coaching session per theme of MRWC during their first implementation year. In addition to that, coaches, along with the teachers, participate in professional learning and collaboration meetings throughout the first implementation year.

The logic model table on the following page describes the program inputs, activities, and instructional practices as well as short-term and long-term outcomes of the MRWC program.

Table 1. MRWC Program Components and Outcome

Mathematical Reasoning with Connections (MRWC i3 grant)

Theoretical basis: Mathematical habits of mind (Cuoco et al, 1996), growth mindset (Dweck, 2006), cognitive demand in effective instruction (AERA, 2006), teacher & student self-efficacy (Tschannen-Moran, Hoy, 2001).

PROGRAM INPUTS

MRWC Professional Development

Teacher

Initial (15 days)

- Five days in the spring of the year preceding implementation
- Ten-day (over 2 weeks) summer institute

Ongoing

- Five days of professional learning and collaboration throughout the implementation year
- Coaches /Teachers on Special Assignment (TSA) will provide support via the coach/mentor model at least 4 times per year (once per module)

Coach

- Five days of PD in the spring of the year preceding implementation.
- Ten day summer institute
- Participation in professional learning and collaboration meetings throughout the implementation year

Principals and Counselors

MRWC Curriculum

Three curriculum modules:

Theme 1 – Reasoning with Numbers

Theme 2 – Reasoning with Functions

Theme 3– Reasoning with Equivalences

MRWC Materials

For Teachers

- MRWC curriculum binders (1 per module)
- MRWC planning guide templates for each participating district/site team.
- End-of-year assessment items

For Students:

- Student binders, handouts,
- MRWC student game sets

INSTRUCTIONAL PRACTICES/CLASROOM ENVIRONMENT

Teachers

- Instruction continuously reiterates commonalities between and among all topics so that students come to understand the entire breadth of high school mathematics curriculum as a cohesive body of knowledge.
- Teachers present students with continuous, consistent, and regular opportunities to discuss and analyze alternative solutions for problems to enhance flexibility with the applications of procedures.
- Classroom activities designed to model and foster real life applications of the advanced mathematical concepts.
- Student-focused activities will promote exploratory and collaborative student engagement.

Students

- Students exposed to more college and career-aligned content and content needed to be successful in college-level mathematics courses.
- Students continuously engage in reflective thinking and reasoning about why the rules are the way they are and the constraints under which rules operate.
- Students learn to apply advanced mathematical concepts to real-life situations.

SHORT TERM OUTCOMES

School/Teacher Outcomes:

- MRWC teachers demonstrate higher levels of pedagogical skills, math teaching efficacy and flexibility.
- Principals and counselors in MRWC schools acquire higher understanding of standards of mathematical practice & CCSS-M

Student Outcomes

- Students increase in mathematical competency and efficacy and learning motivation toward advanced math.
- Student engagement with advanced math curriculum will improve.
- Students completing MRWC course with a passing grade demonstrate math competency equivalent to "college ready" EAP 4 status.

System and Policy Outcomes:

- Increased awareness of need for a 4th year math course aligned to CCSS-M and heightened interest among K-12 and post-secondary institutions for adoption of a 4th year math course to satisfy graduation and A-G (K-12) requirements.

LONG TERM OUTCOMES

School/Teacher Outcomes:

- MRWC teachers will utilize new pedagogy across their courses.
- Subset of MRWC teachers will become certified MRWC trainers and help disseminate the pedagogy.

Student Outcomes

- Students completing MRWC course with a passing grade, will be more successful in their freshman year college-level mathematics courses.
- Students completing the MRWC course with a passing grade will attain higher long-term performance in college courses requiring mathematics (e.g., STEM field courses).

System and Policy Outcomes:

- Student passing grade in MRWC is accepted in post-secondary institutions as "college ready".
- Substantial reduction will be seen in student placement into remedial math at post-secondary institutions.
- Reduction in the use of math entrance exams will be seen throughout the institutions of higher education in California
- Institutions of higher education will adopt the MRWC course to satisfy the fourth year of mathematics/quantitative reasoning for entering first-year students.

Chapter 3. Impacts Analysis of the MRWC Course

This chapter presents the results of the evaluation of the MRWC course and its impact on participating students' mathematics achievement. First, the evaluation methodology is described in detail, followed by data collection procedures and a description of the study sample. Study outcome measure, as well as baseline equivalence analysis are discussed next. The chapter concludes with the discussion of the overall findings of the impact evaluation.

Impact Evaluation Methodology

The impact study used a quasi-experimental design to assess the impact of MRWC on student advanced mathematics achievement and college mathematics readiness skills during the three years of full program implementation (2018-19, 2020-21 and 2021-22). A quasi-experimental design was selected as the most rigorous design possible given the constraints of equal access and voluntary site/district participation. The study compared students who chose to enroll in the 4th year MRWC course taught by MRWC teachers in the schools implementing MRWC (treatment schools) with a matched group of students in advanced mathematics courses taught by teachers not trained on MRWC in the schools not implementing the MRWC in the same school year (comparison schools). The impact study examined the differences in the treatment and comparison group performance on the end-of-year Precalculus Concept Assessment (PCA). A total of about 30-45 treatment schools and up to 6,000 MRWC students were expected to participate in the treatment condition and approximately 15-24 comparison schools with approximately 1,800 students were expected to be included in the study. Treatment students were matched with comparison students within the same year cohort (2018-19, 2020-21 or 2021-22) on several demographic variables including: ethnicity, gender and Grade 11 Mathematics grades using propensity score matching. Matching was conducted "with replacement," such that each comparison student could be a match to multiple MRWC students if that comparison student was similar to multiple MRWC students. Baseline Equivalence testing was performed on the final matched sample in all three study years on treatment and comparison students' average Grade 11 mathematics grades to determine standard mean differences between the two groups on the pre-achievement variable. Originally, grade 11 mathematics SBAC scale scores were intended to be used for baseline equivalence testing, however only the 2018-19 cohort had their grade 11 SBAC scores available. Therefore, the average grade 11 mathematics grade was chosen to assess the baseline equivalence as this was a variable under the same outcome domain and could be easily collected from the participating districts. The average grade 11 mathematics grade was calculated by first converting letter grades earned in each semester of Grade 11 mathematics to a numeric scale and then averaging the numeric values. The numeric scale used was as follows: "A" = 4.0, "A-" = 3.67, "B+" = 3.33, "B" = 3.0, "B-" = 2.67, "C+" = 2.33, "C" = 2.0, "C-" = 1.67, "D+" = 1.33, "D" = 1.0, "D-" = 0.67, "F+" = 0.33, and "F" = 0.0. The baseline equivalence was performed on the final matched sample. Results of the baseline equivalence analysis indicated that between matched MRWC and non-MRWC students, there was a standardized

mean difference of 0.041 in Grade 11 average math grades. As a result, it was concluded that baseline equivalence was achieved, and no additional statistical adjustment was required.

The primary outcome used in the impact analysis study was Precalculus Concept Assessment (PCA), which is a 25-item multiple choice assessment that determines high-school students' readiness for college-level calculus. The test was administered to treatment and comparison students in the Spring of their 4-th year. After matching was complete all of the matched students were included in an ordinary least squares (OLS) regression model that included the same variables as covariates that were used in the matching process (ethnicity, gender, grade 11 mathematics grades). This was done to make the evaluation more robust in that the matching and the regression protects against misspecification in either model (Imbens & Wooldridge, 2009).

Control Conditions

The control group consisted of students taking traditional 4th year advanced mathematics courses in classrooms taught by teachers who have not participated in any MRWC training during their participation in the control condition. Comparison teachers operated under "business-as-usual" and did not receive any additional technology, PD, or coaching related to MRWC in the year/s that they were participating in the comparison condition. Students in comparison classrooms received the typical "business-as-usual" year 4 mathematics curriculum, texts and materials. Implementation and program inputs were monitored closely and compared across groups to track the potential impact of confounding variables.

Most teachers who served as comparison teachers for one year were given an opportunity to be trained as an MRWC teacher in the following year, if their district and site elected to participate and could commit to creating a team (2 mathematics teachers, instructional coach, administrator, and counselor) to be trained on MRWC. Therefore, the study followed a waitlist control model to allow the comparison schools a chance to implement the MRWC course after engaging in the control condition.

Sample Selection and Assignment

All of the students included in the treatment and comparison samples were high school students taking an advanced mathematics course enrolled in high schools in Inland Empire during the 2018-2019, 2020-2021 and 2021-22 school years. A total of 45 treatment schools from multiple Riverside and San Bernardino County school districts with 5,415 MRWC students were included into the final treatment sample and 19 comparison schools with 1,462 students were included in the study.

Inferences from this study could be generalized to high-school students in need of fourth-year mathematics course to be considered ready for college-level mathematics courses from large and mid-size urban, suburban and rural school districts with significant Hispanic, socio-economically disadvantaged, and English Learner populations.

Selection of Study Schools

Treatment schools were recruited from the local (Inland Empire) school districts by disseminating information about the new fourth-year mathematics course to Inland Empire districts and offering the interested districts to include the MRWC course on the master schedule and put together a team consisting of two high school mathematics teachers, a mathematics instructional coach, a high school counselor and high school administrator (principal or assistant principal) and agree for that team to be trained on delivering the MRWC course. The school teams that expressed an interest in participating in the MRWC study entered into a Memorandum of Understanding (MOU) with the project management team that outlined the rights and responsibilities of participating parties and was signed by project management team and the external evaluator as well as participating district teams (teachers, coaches, and principals).

Once participating teams were identified and trained on MRWC course delivery, MRWC course was put on the participating schools master schedules. Schools were recruited into the treatment group if they met the following criteria: 1) the school must provide a team of teachers, a coach, a principal and a counselor to be trained in and implement MRWC course and commit to at least one year of MRWC implementation, 2) the teachers must be willing to participate in the study by teaching the MRWC course, completing the teacher surveys and fidelity implementation reflections and logs and administering PCA assessment and student surveys to students who were enrolled in and completed their MRWC course, and 3) the school must be willing to provide student-level data on prior achievement and demographics variables. Up to 15 treatment schools were recruited each year and were able choose to continue program implementation for subsequent study years. In study years 2 and 3, new treatment schools were recruited from the pool of comparison schools from the previous year as well as other similar schools.

Comparison group schools were recruited from the neighboring districts that were not currently implementing the MRWC course by first identifying the local school districts and schools that would agree to serve as comparison schools for a period of at least one year. After serving as comparison schools for one year, the schools were offered to become treatment sites, provided they were willing to implement the MRWC course and could assemble a team that included two teachers and a mathematics coach. Comparisons schools that elected not to implement the MRWC course could continue serving as control sites. Schools were recruited into the comparison group using the following criteria: 1) school administrators agreed to provide access to at least two teachers who teach advanced mathematics course (e.g., Statistics and Probability, Pre-Calculus, Integrated Math 4, etc.) and students enrolled in that course, 2) schools agreed to provide student-level data on demographics and prior (grade 11) mathematics performance of students currently enrolled in those fourth-year mathematics courses 3) teachers agreed to administer PCA and student surveys to all of the students enrolled in their advanced mathematics courses and complete teacher perception surveys at the end of spring. Up to eight comparison schools were included each year.

Selection of Study Teachers

Once treatment MRWC schools were identified and agreed to participate, they were asked to put together a team to be trained on MRWC course delivery. This team included at least two high school mathematics teachers that have taught at least one advanced mathematics course before (e.g., Pre-Calculus, Calculus, Statistics), a high-school mathematics coach, a counselor and an administrator. Between 15-30 teachers were recruited and trained on MRWC course delivery in each of the implementation cohorts.

Comparison schools were asked to identify at least two teachers that teach at least one advanced mathematics course (beyond Algebra II or IM3) to serve as comparison teachers. Once comparison teachers agreed to participate, they were included in the study. These teachers delivered their traditional advanced mathematics courses without receiving any additional professional development or materials related to MRWC. Comparison teachers were not expected to have any interactions with the MRWC teachers as they were in different schools.

Selection of Study Students

The study was conducted with three cohorts of high school students taking a fourth-year mathematics course in 2018-19, 2020-21 and 2021-22. Once participating teacher teams were identified and trained on MRWC course delivery, MRWC course was added to each of the schools' master schedules and high school counselors disseminated course information (flyers containing course description and purpose, description of the study and consent and assent forms) to eligible students who were interested in taking a fourth-year mathematics course in participating high schools.

Since MRWC course has been specifically designed to address the need for stronger mathematics preparation for transitioning from high school to college and career pathways and reduce the need for students to enroll in remedial mathematics courses upon entering college, the study focused on high-school students who need a fourth-year mathematics course to be considered ready for college-level mathematics. The following criteria were originally proposed to identify students for the participation in this study for both treatment and control conditions:

- a minimum grade of C in Integrated Math 3 or both Algebra 2 and Geometry
- SBAC Level 2 (Not Yet Ready), SBAC Level 3 (Conditionally Ready)
    - this rule was only applicable to the 2018-19 cohort due to the SBAC state testing cancellation for the later cohorts
- Interested in taking a fourth-year mathematics course

Comparison students were recruited from the advanced mathematics classes in comparison schools taught by comparison teachers identified as described in the previous sections. Comparison students all took a year-long traditional advanced mathematics course (Precalculus, Statistics and Probability, Integrated Math 4, etc.).

Comparison group students were matched with treatment group students within the same year cohort on several demographic variables including ethnicity, gender, and Grade 11 average Mathematics grades. Using logistic regression, propensity scores for all students (both MRWC and comparison) were estimated based on the set of matching variables. The distribution of propensity scores in the treatment group and the comparison group was compared to see if the two groups span a similar range (i.e., have similar propensity to enroll in MRWC). We divided the propensity scores into quintiles to create five strata. Comparison group students were matched to treatment group students in the same stratum. Matching was done with replacement. Given that the number of students in the treatment group is larger than the number of students in the comparison group, this approach to matching will maximize the total sample size.

Sample Sizes

Participating study districts were located in Inland Empire (Riverside and San Bernardino Counties). Thirty-one districts were involved in the final implementation study. Within those districts, there were forty-five intervention schools and nineteen comparison schools. The final unmatched sample sizes per cohort are shown below in tabular form.

Table 2. Final Student Level Sample Sizes Before Matching

	Intervention Group	**Comparison Group**
2018-19 MRWC Cohort	2,153	516
2020-21 MRWC Cohort	1,322	472
2021-22 MRWC Cohort	1,940	674

Table 3. Final Student Level Sample Sizes After Matching

	Intervention Group	**Comparison Group**
2018-18 MRWC Cohort	2,153	2,149
2020-21 MRWC Cohort	1,322	1,322
2021-22 MRWC Cohort	1,940	1,924

Data Collection

The data collection matrix below presents the annual schedule for data collection. Data collection was repeated on this schedule for each year of the impact study.

Table 4. MRWC Data Collection Timeline

MRWC Instruments Data Collection Timeline (Outcome)		
Data	Sample	When
PCA Mathematics Test (outcome variable)	TS, CS	EOY

Pre-existing Data Collection Timeline (Independent variables)		
Data	Sample	When
Student Ethnicity	TS, CS	BOY
Student Gender	TS, CS	BOY
Grade 11 SBAC Scale Score	TS, CS	BOY (18-19 only)
Grade 11 math grades (baseline measure)	TS, CS	BOY

TS = treatment students; CS = comparison students; BOY = Beginning of Year (pre); EOY = End of Year (post)

Dependent Variable (Student Outcome)

Outcome Domain: Mathematics Achievement and College Readiness Skills

Outcome Measure: Precalculus College Assessment

Dependent variable was selected for its reliability, validity as well as its alignment with the goals and objectives of the program. Key outcome domain evaluated in this study was high school students' mathematics achievement and college readiness skills and the primary outcome measure that was used in the impact study was the students' performance on the Precalculus Concept Assessment (PCA). This assessment is a 25-item multiple choice test that assesses a broad taxonomy of reasoning abilities, ideas of function, function composition, function notation, rate of change, exponential growth and other topics addressed across various high school advanced mathematics curricula. A number of validation studies conducted for this assessment found that it can be useful in assessing the effectiveness of advanced high school mathematics courses in preparing student to be successful in college calculus (Carlson, Oehrtman & Engelke, 2010; Lindley, 2021). The test was administered to treatment and comparison students in the Spring of their participation year after completing the course. PCA was administered to the students during their regular class time at the end of their MRWC course (treatment students) or their traditional mathematics course (control students). The PCA scores are reported as percent correct. For the impact analysis, this variable was used as a continuous variable and standardized mean differences between treatment and comparison group on this variable were examined.

Independent Variables

MRWC enrollment

The primary independent variable was program participation (intervention vs. comparison). Students at the intervention schools experienced the full program, while comparison students experienced, "business-as-usual" traditional advanced mathematics courses.

Student ethnicity/race

This was a categorical variable used to determine the extent to which students' mathematics achievement depends on their ethnicity/race. Student ethnicity categories used in the analysis were African American, Asian, Hispanic and White. Student ethnicity/race information was obtained by requesting enrollment files from participating school districts.

Student gender

This categorical variable (1- Female, 0 – Male) was used to determine the extent to which students' mathematics achievement depends on students' gender. Student gender status was obtained by requesting enrollment files from participating school districts.

Baseline Measure: Grade 11 Average Mathematics Grade
Baseline Measure Domain: Mathematics Achievement

Originally, grade 11 mathematics SBAC scale scores were intended to be used for baseline equivalence testing, however only the 2018-19 cohort had their grade 11 SBAC scores available. The 2020-21 cohort did not take the SBAC test in 2019-20 due to distance learning and SBAC testing cancellation across California. The 2021-22 cohort had SBAC scores available only for some of the students, as 2020-21 SBAC testing participation was not mandatory for the districts and only about 40% of the enrolled grade 11 students participated in SBAC math testing that year. Therefore, the average grade 11 mathematics grade was chosen to assess the baseline equivalence as this was a variable under the same outcome domain and could be easily collected from the participating districts.

The average grade 11 mathematics grade was calculated by first converting letter grades earned in each semester of Grade 11 mathematics to a numeric scale and then averaging the numeric values. The numeric scale used was as follows: "A" = 4.0, "A-" = 3.67, "B+" = 3.33, "B" = 3.0, "B-" = 2.67, "C+" = 2.33, "C" = 2.0, "C-" = 1.67, "D+" = 1.33, "D" = 1.0, "D-" = 0.67, "F+" = 0.33, and "F" = 0.0. Files with prior year (grade 11) math course grades from both semesters were requested from participating school districts by the external evaluator.

Contrast

The impact analysis examined the effectiveness of the new fourth year Mathematical Reasoning with Connections (MRWC) course for preparing students to enter college-level mathematics courses without remediation. The main question examined in this impact evaluation study was as

follows: Does Mathematical Reasoning with Connections senior year mathematics course increase the mathematics achievement of senior year high school students, compared to senior year high school students enrolled in business-as-usual senior year mathematics courses.

Baseline Equivalence Testing

Baseline equivalence testing is necessary to determine whether the sample of MRWC students is similar to the sample of non-MRWC students included in the analysis. This testing is conducted on the final analytic sample after the matching has been performed. The prior achievement variable was tested for baseline equivalence using the final matched analysis samples.

In accordance with A & R Team recommendations, the difference in means between the intervention and comparison group was calculated and divided by the pooled SD, to compute the effect size of baseline difference. If the difference between the intervention and comparison mean value for the baseline is less than or equal to .05 SD, it could be concluded that equivalence has been established. If the SD is between .05 - .25 and the variable was included in the regression model, it could be concluded that equivalence has been established. If the difference in SD between the intervention and comparison group is greater than .25, it should be concluded that baseline equivalence has not been established.

Table 5 provided below shows the results of the baseline equivalence analysis. As shown in table, between matched MRWC and non-MRWC students, there was a standardized mean difference of 0.041 in Grade 11 average math grades. As a result, it was concluded that baseline equivalence was achieved, and no additional statistical adjustment was required.

Table 5. Baseline Equivalence Testing

A	B	C	D	E	F	G	J	K
Contrast ID #	Contrast Name (optional)	Baseline Measure Name	Treatment Group N	Comparison Group N	Unadjusted Treatment Group Mean	Unadjusted Comparison Group Mean	Treatment – Comparison Difference	Absolute Effect Size of Baseline Differences (Hedge's g)
1	Student Mathematics Achievement	Grade 11 Mathematics Grades	5,415	5,393	2.596	2.670	0.074	**0.041**

Statistical Analysis of Impacts on Students

Model Specifications

Impact analysis was conducted using an ordinary least squares (OLS) regression model that included the same variables as covariates that were used in the matching process. This was done to make the evaluation more robust in that the matching and the regression protects against misspecification in either model (Imbens & Wooldridge, 2009).

RQ1: What is the effect of MRWC on students' mathematics achievement as measured by PCA assessment compared to that of students enrolled in business-as-usual senior year mathematics courses? (Confirmatory).

Impact Analysis Model

$$Y_i = \alpha + \beta 1\ (MRWC_i) + \beta 2\ (Math\_Grade_i) + \mathbf{\beta 3\ (Ethnicity_i)} + \beta 4\ (Female_i) + \varepsilon_i$$

where:

Y_i is the PCA score for student i;

α is the intercept;

MRWC is a binary variable, indicating MRWC (1) or control (0) student

Math_Grade is the average Math grade earned in Grade 11

Ethnicity is a vector of dichotomous variables indicating ethnicity for student i

(African American, Asian, Hispanic, or White);

Female is a binary variable identifying female students;

ε is the error term.

In this model, β_1- β_4 are parameters to be estimated from the data, and ε is the error term. β_1 in the above regression equation represents the average difference in PCA scores between MRWC and non-MRWC students after controlling for the covariates included in the model; this parameter represents the impact of the MRWC curriculum. In other words, it represents the average difference in outcomes between students in MRWC and non-MRWC courses after controlling for the covariates in the model. The hypothesis test for β_1 will determine whether or not the intervention has a statistically significant impact on the given outcome.

Table 6 displays the results of the OLS analysis that included the matched MRWC and control students. The estimate for the MRWC enrollment represents the impact for the MRWC. The estimate is positive and statistically significant at the 1 percent level. This is indicative that in this impact study MRWC was found to have a positive impact on student mathematics achievement, and that this impact was unlikely to have happened by chance.

Table 6. Regression Analysis Results/Impact of MRWC on Precalculus Concept Assessment

A	B	C	D	E
Characteristic	**Estimate**	**Standard Error**	**t-Statistic**	**p-value**
Intercept	27.983	.512	54.625	.000
MRWC enrollment	**4.187**	.261	16.123	**<.001**
Average Grade 11 Math Grade	2.316	.416	4.009	<.001
Asian	2.483	.611	4.061	<.001
Hispanic	-2.714	.396	-6.846	<.001
White	-.282	.491	-.576	.565
African American	-1.434	.592	-2.423	.015
Female	-0.583	.264	-2.207	.027

Impact Estimates and Effect Sizes

Hedges' g was used to calculate effect sizes to measure the magnitude of the program's primary effects, as this is a suggested measure with small effect sizes (What Works Clearinghouse, 2020). Estimates were calculated from the data and a pooled SD was used.

Table 7 displays the estimated effect size of the MRWC. In calculating the effect size, the adjusted mean difference from the regression model was used as suggested by Lipsey et al., (2012). Taking MRWC course was associated with a small positive (0.31 or 31% of a standard deviation) treatment effect on students' mathematics achievement measured by the Precalculus Concept Assessment.

Effect Size Considerations

Previously published reports suggested that effect sizes from the studies of educational interventions should be interpreted in context of the key elements of the intervention, as well as the population involved. Specifically, the type of the outcome measures used, population involved in the study and the type of intervention should be considered when interpreting the relative magnitude of effect sizes (Fong, Finkelstein, Jaeger, Diaz & Broek, 2015; Lipsey et al., 2012). For example, Lipsey et al. (2012) suggested that effects sizes for standardized tests that measure broad subject matter (e.g., SBAC Math) tend to be the smallest, with a mean of 0.07. Specialized tests, developed to measure specific skills (e.g., PCA) tend to have larger average effect sizes (e.g., 0.44). Such studies suggest that the effect sizes of an intervention measured on the broad standardized test will be smaller, while the effects on the assessments created to measure more specific skills will be larger. Our study used a specialized assessment that measured advanced mathematics skills, so our study expected higher effect sizes than if we used standardized mathematics assessment, such as SBAC Math.

With regard to intervention format, Lipsey et al. (2012) reported that mean effect sizes differ depending on the type of the intervention being implemented. Specifically, curriculum or broad-instructional program interventions, such as i3 MRWC intervention have one of the smallest average effect sizes (0.13), potentially due to their broad focus. Considering this, the effects of the MRWC course on students' mathematics skills should be considered substantial in the context of this particular intervention type.

Table 7. Impact Estimate and Effect Sizes

A	B	C	D	E	F	G	H	I	G	H	I
Contrast ID #	**Contrast Name (optional)**	**Post-test Measure Name**	**Treatment Group N of Students**	**Unadjusted Treatment Group Mean**	**Unadjusted Treatment Group SD**	**Comparison Group N of Students**	**Unadjusted Comparison Group Mean**	**Unadjusted Comparison Group SD**	**Pooled Standard Deviation**	**Impact Estimate**	**Estimated Effect Size**
1	Student Mathematics Achievement	**Precalculus Concept Assessment** [Continuous]	5,415	30.29	14.37	5,393	26.07	12.06	13.52	**4.187**	**0.311**

Chapter 4. Evaluating Implementation and Assessing Fidelity

The MRWC Program included three components: mathematics curriculum and strategies, MRWC student and teacher materials and professional learning. Fidelity tables were created to collect data on and determine whether these three components were implemented with fidelity.

Implementation research questions were developed to assess the fidelity of implementation of each component. Fidelity of implementation analysis was guided by the following questions:

1. What proportion of study teachers are implementing the MRWC course with adequate fidelity as measured by MRWC teacher implementation logs and reflections and coaching observation forms?

2. What proportion of MRWC teachers received coaching observations at least three times during the implementation year (once per theme taught)?

3. What proportion of MRWC teachers attended at least 13 out of the 15 required pre-implementation professional development days?

4. What proportion of MRWC teachers attended at least four out of five days of professional learning and collaboration throughout the implementation year (5 days)?

5. What proportion of study teachers received all of the required curriculum materials (teacher and student) prior to teaching each respective module?

Additional exploratory fidelity question examined:

1. How many MRWC teachers completed the MRWC train-the-trainer certification workshop and become certified MRWC trainers by the study close?

The data on implementation fidelity were collected in the following manner:

1. Professional learning participation was tracked using a professional learning database. Sign-in sheets at each training were logged into the system to track attendance at trainings and workshops to ensure that intervention teachers are receiving the required number of professional development hours.
2. Coaching sessions were recorded via coaching reflections filled out by the coaches after every visit. Coaching sessions were then logged into the system to track collaboration opportunities.
3. Teaching fidelity was tracked using an online teaching reflection tool that has been created by the project's external evaluator to measure the level of implementation within each classroom. Teachers rated implementation of each activity within a particular Theme on a four-point scale (e.g., not completed, partial completion, substantial completion, and full/ in-depth completion). For each activity that was not completed or partially completed teachers

were asked to provide a reason. Total scores per teacher were calculated with high scores indicating higher levels of implementation. Results were used to provide formative feedback to the program with regards to implementation fidelity. Results were also used in the annual performance reports to assist in the interpretation of outcomes.

4. Course materials were created by the curriculum and leadership team, assembled into student and teacher binders and shared with each teacher. Games, pacing guides and accompanying sample assessment items for each of the Themes were also shared with the participating teachers. The project support personnel tracked the delivery of materials and shared with the project evaluator.

Measuring Fidelity of Implementation

Fidelity tables were created to present the levels of fidelity to identify how well the MRWC program was implemented. Fidelity table is presented on the following page. The components listed on the furthest column on the left were evaluated by teacher and overall, across sites. First, fidelity scores were calculated for each teacher. Then, sample-level fidelity was calculated, combining all of the teachers (sample-level fidelity). Each teacher received a fidelity score for each key component. We also calculated a comprehensive program level fidelity score across all MRWC teachers for each component. This gave us a good indication of how well the program was being implemented each year of the grant.

The following table presents the implementation fidelity scores by the implementation year for all of the intervention implementation components. The results of the fidelity of implementation analysis indicated that all five of the fidelity of implementation components were implemented with fidelity in all three of the years that the fidelity implementation study was conducted in.

Additional exploratory fidelity question had to do with providing the train-the-trainer certification and using MRWC techniques in other courses. Regarding the first component, in the beginning of 2019, the MRWC project management team completed the development of the train-the-trainer (ToT) professional development course and certification. With the goal of training a total of 20 apprentices across the last 2 years of the grant, this target was evenly divided to create an annual target of 10 apprentices trained in each of the last two years of the grant. Eleven train-the-trainer apprentices were recruited and have completed their training with the first cohort in 2019. In 2020, 8 apprentices also completed the process. One additional apprentice finished the training in February of 2021. Additionally, we have trained 8 more apprentices during our final (no-cost extension) year (2022). The total number of MRWC ToT teacher leaders trained over the grant period was 28.

Table 8. Implementation Fidelity Results

Fidelity of Implementation of Intervention(s) by Year

	Findings from Evaluation Study of Implementation: IMPLEMENTATION YEAR 1 **School Year: 2018-19**						
Intervention Components:	**Sample Size at the Sample Level** (e.g., # of teachers)	**Representativeness of sample: Measured on All (A), Some (S), or None (N) of the units**	**Score for levels of implementation at unit level**	**Component Level Threshold for Fidelity of Implementation for the Unit that is the Basis for the Sample-Level**	**Evaluator's Criteria for "Implemented with Fidelity" at Sample Level**	**Component Level Fidelity Score for the Entire Sample**	**Implemented with Fidelity?** (Yes, No, N/A)
Key Component 1: MRWC Implementation							
Implementation of MRWC strategies and course curriculum	17	A	Amount of curriculum taught with adequate fidelity: 0 - 0-65% (low) 1 – 66-84% (moderate) 2 – 85-100% (high)	2 = individual teachers teach at least 85% of the curriculum with fidelity	Adequate sample fidelity = at least 80% of the MRWC teachers score 2	94%	yes
Key Component 2: MRWC Professional Learning							
Subcomponent 1: Teacher participation in professional learning before implementation (15 days)	17	A	Days attended: 0 (low) – attended 0-7 days 1 (moderate) - attended 8-12 days 2 (high) – attended 13-15 days	Teacher level fidelity threshold – score of 2	Adequate fidelity = at least 85% of MRWC teachers score 2	100%	yes
Subcomponent 2: Teacher participation in professional learning during implementation (5 days)	17	A	Days attended: 0 (low) – attended 0-1 days 1 (moderate) - attended 2-3 days 2 (high) – 4-5 days	Teacher level fidelity threshold – score of 2	Adequate fidelity = at least 85% of MRWC teachers score 2	100%	yes
Subcomponent 3: Coaching observations (3 observations/year)	17	A	Number of coaching sessions: 0 (low) – zero to one coaching sessions 1 (moderate) – two coaching sessions 2 (high) – three coaching sessions per year	Teacher level fidelity threshold – score of 2	Adequate fidelity = at least 85% of teachers score 2	100%	yes
Key Component 3: MRWC Materials							
MRWC Materials	17	A	0- did not receive all of the materials on time 1 – received all of the materials on time	Teacher level with adequate fidelity – score of 1	Adequate fidelity = at least 95% of MRWC teachers score 1	100%	yes

	Findings from Evaluation Study of Implementation: IMPLEMENTATION YEAR 2 **School Year: 2020-21**						
Intervention Components:	**Sample Size at the Sample Level** (e.g., # of teachers)	**Representativeness of sample: Measured on All (A), Some (S), or None (N) of the units**	**Score for levels of implementation at unit level**	**Component Level Threshold for Fidelity of Implementation for the Unit that is the Basis for the Sample-Level**	**Evaluator's Criteria for "Implemented with Fidelity" at Sample Level**	**Component Level Fidelity Score for the Entire Sample**	**Implemented with Fidelity?** (Yes, No, N/A)
Key Component 1: MRWC Implementation							
Implementation of MRWC strategies and course curriculum	17	A	Amount of curriculum taught with adequate fidelity: 0 - 0-65% (low) 1 – 66-84% (moderate) 2 – 85-100% (high)	2 = individual teachers teach at least 85% of the curriculum with fidelity	Adequate sample fidelity = at least 80% of the MRWC teachers score 2	87%	yes
Key Component 2: MRWC Professional Learning							
Subcomponent 1: Teacher participation in professional learning before implementation (15 days)	25	A	Days attended: 0 (low) – attended 0-7 days 1 (moderate) - attended 8-12 days 2 (high) – attended 13-15 days	Teacher level fidelity threshold – score of 2	Adequate fidelity = at least 85% of MRWC teachers score 2	100%	yes
Subcomponent 2: Teacher participation in professional learning during implementation (5 days)	25	A	Days attended: 0 (low) – attended 0-1 days 1 (moderate) - attended 2-3 days 2 (high) – 4-5 days	Teacher level fidelity threshold – score of 2	Adequate fidelity = at least 85% of MRWC teachers score 2	100%	yes
Subcomponent 3: Coaching observations (3 observations/year)	25	A	Number of coaching sessions: 0 (low) – zero to one coaching sessions 1 (moderate) – two coaching sessions 2 (high) – three coaching sessions per year	Teacher level fidelity threshold – score of 2	Adequate fidelity = at least 85% of teachers score 2	100%	yes
Key Component 3: MRWC Materials							
MRWC Materials	17	A	0- did not receive all of the materials on time 1 – received all of the materials on time	Teacher level with adequate fidelity – score of 1	Adequate fidelity = at least 95% of MRWC teachers score 1	100%	yes

	Findings from Evaluation Study of Implementation: IMPLEMENTATION YEAR 3 **School Year: 2021-22**						
Intervention Components:	**Sample Size at the Sample Level** (e.g., # of teachers)	**Representativeness of sample: Measured on All (A), Some (S), or None (N) of the units**	**Score for levels of implementation at unit level**	**Component Level Threshold for Fidelity of Implementation for the Unit that is the Basis for the Sample-Level**	**Evaluator's Criteria for "Implemented with Fidelity" at Sample Level**	**Component Level Fidelity Score for the Entire Sample**	**Implemented with Fidelity?** (Yes, No, N/A)
Key Component 1: MRWC Implementation							
Implementation of MRWC strategies and course curriculum	25	A	Amount of curriculum taught with adequate fidelity: 0 - 0-65% (low) 1 – 66-84% (moderate) 2 – 85-100% (high)	2 = individual teachers teach at least 85% of the curriculum with fidelity	Adequate sample fidelity = at least 80% of the MRWC teachers score 2	88%	yes
Key Component 2: MRWC Professional Learning							
Subcomponent 1: Teacher participation in professional learning before implementation (15 days)	25	A	Days attended: 0 (low) – attended 0-7 days 1 (moderate) - attended 8-12 days 2 (high) – attended 13-15 days	Teacher level fidelity threshold – score of 2	Adequate fidelity = at least 85% of MRWC teachers score 2	100%	yes
Subcomponent 2: Teacher participation in professional learning during implementation (5 days)	25	A	Days attended: 0 (low) – attended 0-1 days 1 (moderate) - attended 2-3 days 2 (high) – 4-5 days	Teacher level fidelity threshold – score of 2	Adequate fidelity = at least 85% of MRWC teachers score 2	100%	yes
Subcomponent 3: Coaching observations (3 observations/year)	25	A	Number of coaching sessions: 0 (low) – zero to one coaching sessions 1 (moderate) – two coaching sessions 2 (high) – three coaching sessions per year	Teacher level fidelity threshold – score of 2	Adequate fidelity = at least 85% of teachers score 2	100%	yes
Key Component 3: MRWC Materials							
MRWC Materials	25	A	0- did not receive all of the materials on time 1 – received all of the materials on time	Teacher level with adequate fidelity – score of 1	Adequate fidelity = at least 95% of MRWC teachers score 1	100%	yes

Chapter 5. Conclusion and Discussion

This report provides the results from a three-year evaluation of the i3 MRWC program and its impact on student mathematics achievement and college readiness across Inland Empire. One hundred and forty-four schools implemented MRWC in their advanced mathematics classrooms over the last five years. Mathematics performance of the students that were exposed to the three intervention components of the MRWC program: curriculum and strategies, professional learning and MRWC materials was compared to the performance of students in comparison classrooms that took a traditional advanced mathematics course and that did not receive the MRWC curriculum. Students in the control classrooms were matched to students in the intervention schools on a number of demographic variables and prior mathematics achievement. The outcome measure used to examine the impact of the MRWC was Precalculus Concept Assessment.

Results of the impact analysis indicated that students in the i3 classrooms scored significantly higher than the control students on the PCA. The effect size surpassed the cutoff that the What Works Clearinghouse set for the substantive positive effect sizes. The cut-off is 0.25SD and the effects obtained on student mathematics outcome was 0.31SD for the effect of one year of exposure to MRWC on high school students mathematics achievement.

Implementation study was also conducted, using attendance logs (professional learning, coaching observation, and collaboration), fidelity implementation logs and reflections and material delivery logs. All of the fidelity components were implemented with high fidelity in all three years of the study.

The impact evaluation analysis found that MRWC produced a small but significant impact on student mathematics achievement and readiness for college-level calculus and the effect sizes can be considered substantial as it should be noted that as a district-wide curriculum intervention, the MRWC effect sizes were comparable to if not slightly higher than the average effect sizes produced by other studies of district-wide curriculum interventions reviewed by Lipsey et al. (2012). Additionally, the participating Inland Empire teachers, administrators and counselors consistently shared with the project leadership that both teachers and students involved definitely enjoyed many benefits of the program and that this intervention still has a potential to improve students' mathematics achievement, especially if students are exposed to these strategies as early as possible in their high school mathematics experience. Therefore, future evaluations could assess whether the impacts of this pedagogy and professional learning model would be even greater, if teachers were able to implement it throughout all of the high school mathematics courses they teach.

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